

On first Zagreb energy of total dominating set in graphs*

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Abstract. Let G be a simple graph of order n with vertex set $V(G)$ and edge set $E(G)$. Let MTD be the minimum total dominating set of G and $|MTD| = \gamma_t(G)$ the total domination number. The first Zagreb matrix of minimum total dominating set of G is the $n \times n$ matrix defined by $Z_{MTD}(G) = (z_{ij})$ where

$$Z_{MTD}(G) = \begin{cases} d_i + d_j, & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 1, & \text{if } i = j \text{ and } v_i \in MTD \\ 0, & \text{otherwise.} \end{cases}$$

where d_i and d_j are the degree of the vertex v_i and v_j ($1 \leq i < j \leq n$), respectively and v_1, v_2, \dots, v_n are the vertices of the $n \times n$ graph G . The first Zagreb energy of total dominating set of G , denoted by $ZE_{TD}(G)$, is the summation of the *eigenvalues* of G where the *eigenvalues* of G are the zeros of the characteristic polynomial of $Z_{MTD}(G)$. In this study, we determine the exact values or bounds of $ZE_{TD}(G)$ of some special graphs. Moreover, we also provide some characterizations for $ZE_{TD}(G)$ and its relationship with Zagreb energy $ZE(G)$.

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